

# Gravitational lenses magnify up to one third of the most distant quasars

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Exceptionally bright quasars with redshifts up to  $z=6.28$  have recently been discovered<sup>1</sup>. Quasars are thought to be powered by the accretion of gas onto supermassive black holes at the centres of galaxies. Their maximum (Eddington) luminosity is proportional to the mass of the black hole, and so these bright quasars are inferred to have black holes with masses of more than a few billion solar masses. The existence of such massive black holes poses a challenge to models for the formation of structures in the early Universe, as it requires that the black holes would grow so massive in less than a billion years after the Big Bang. Here we show that up to a third of known quasars with  $z \sim 6$  will have their observed flux magnified by a factor of 10 or more through gravitational lensing by galaxies along the line of sight. The inferred abundance of quasar host galaxies, as well as the luminosity density provided by the quasars, are therefore substantially overestimated.

The four highest redshift quasars known<sup>1</sup> with  $z \gtrsim 5.8$  (SDSS 1044-0125 was later found<sup>4</sup> to have  $z = 5.73$ ), were selected in the SDSS photometric system to have magnitudes  $z^* < 20.2$  and colors  $i^* - z^* > 2.2$ . The masses of the black holes powering these quasars are estimated to be  $\gtrsim 3 \times 10^9 M_\odot$ , implying<sup>3</sup> that the mass of their host galaxies is  $\gtrsim 10^{13} M_\odot$ . Such massive hosts lie on the steep exponential tail of the Press-Schechter<sup>5</sup>

mass function, so that a correction to the inferred black-hole mass severely impacts the estimated cosmological density of galaxy halos which are sufficiently massive to host the observed quasars.

Gravitational lensing leads to a magnification by a factor  $\mu$  of the apparent source luminosity. Let us first consider the implications of this magnification for the inferred properties of the massive quasar systems within structure formation theory. Using the Eddington luminosity to set a lower limit on the inferred black hole mass<sup>3</sup>, one finds that the inclusion of the magnification due to lensing lowers the minimum black hole mass by a factor of  $\mu$ . This, in turn, implies that the black hole can form in a lower mass galaxy. Figure 1 shows the resulting enhancement factor in the space density of galaxy halos that could host the observed SDSS quasars at  $z = 6$ . The inclusion of lensing has a dramatic effect (by orders of magnitude) on the expected abundance of such hosts at early cosmic times. The prominence and duty cycle of such hosts has important implications<sup>7,8</sup> with respect to the question of whether the cosmic neutral hydrogen, a cold remnant from the big bang, was re-ionized by star light or by quasars<sup>9,10</sup>.

To calculate the probability of gravitational lensing in a quasar sample we need to specify the luminosity function (number per comoving volume per unit luminosity) that describes the quasar population. At  $z \lesssim 3$  this function is well described by a double power-law whose shape does not evolve with redshift<sup>11</sup>,

$$\phi(L, z) = \frac{\phi_*/L_*(z)}{[L/L_*(z)]^{\beta_l} + [L/L_*(z)]^{\beta_h}}. \quad (1)$$

The observed evolution of the break luminosity  $L_*$  is described by the dependence<sup>9</sup>

$$L_*(z) = L_*(0)(1+z)^{-(1+\alpha_q)} e^{\zeta z} \frac{1 + e^{\xi z_*}}{e^{\xi z} + e^{\xi z_*}}. \quad (2)$$

We find that an intrinsic luminosity function having the parameters  $\phi_* = 624 \text{ Gpc}^{-3}$ ,  $\beta_l = 1.64$ ,  $\beta_h = 3.43$ ,  $L_*(0) = 1.5 \times 10^{11} L_\odot$ ,  $\alpha_q = -0.5$ ,  $z_* = 1.45$ ,  $\xi = 2.9$ ,  $\zeta = 2.7$ ,

and the inclusion of gravitational lensing (described below) adequately describes three observables, namely the luminosity function at  $z \lesssim 3$ , and the number density of quasars with absolute B-magnitude  $M_B < -26$  at  $z \sim 4.3$  (measured by SDSS from a catalog of 39 quasars<sup>12</sup> with a median redshift of  $z \sim 4.3$ ) and  $M_B < -27.6$  at  $z \sim 6.0$ . The parameter  $\alpha_q$  is the slope assumed for the typical quasar continuum  $L(\nu) \propto L^{\alpha_q}$ . We are interested in the number of quasars with luminosities higher than the limiting magnitude  $z_{\text{lim}}^* = 20.2$ , which is  $N(> L_{\text{lim}}, z) = \int_{L_{\text{lim}}}^{\infty} dL \phi(L, z)$  where  $L_{\text{lim}}$  is the luminosity of a quasar at redshift  $z$  corresponding to an apparent magnitude  $z_{\text{lim}}^*$ .  $L_{\text{lim}}$  was determined from  $z_{\text{lim}}^*$  using a luminosity distance and a  $k$ -correction computed from a model quasar spectrum including the mean absorption by the intergalactic medium<sup>13</sup>.

Gravitational lensing is expected to be highly probable for very luminous quasars<sup>14</sup>. To illustrate this point, we consider a fictitious gravitational lens that always produces a magnification of  $\mu = 4$  for the sum of multiple images [the average value for a singular isothermal sphere (SIS)] but  $\mu = 1$  otherwise. We define  $\tau_{\text{mult}}$  as the probability that a random quasar selected in the source plane will be multiply imaged<sup>15</sup>, and  $F_{\text{MI}}$  to be the magnification biased probability that an observed quasar will be multiply imaged. Surveys for quasars at  $z < 3$  have limiting magnitudes fainter than the break magnitude ( $m_B \sim 19$ ). While  $\tau_{\text{mult}} \sim 0.002$  at  $z \sim 2$ , a survey for quasars to a limit  $L_{\text{lim}}$  will find a number of lensed sources that is larger by a bias factor of  $N(< L_{\text{lim}}/\mu, z)/N(< L_{\text{lim}}, z)$ . At  $z = 2$ ,  $L_{\text{lim}}(z) \ll L_{\star}(z)$  and for  $\beta_l = 1.6$  the magnification bias factor is  $\sim 2.3$ , resulting in  $F_{\text{MI}} \sim 0.005$ . At  $z = 6$ ,  $\tau_{\text{mult}} \sim 0.008$  is significantly higher<sup>16</sup>. Furthermore, the limiting magnitude of the  $z \gtrsim 5.8$  survey is significantly brighter than the break magnitude and so  $\beta_h = 3.4$  and the bias factor rises to  $\sim 28$ . Under these circumstances,  $F_{\text{MI}}$  rises to  $\sim 0.22$ . These simple arguments are consistent with previous estimates of the lensing rate at high redshift<sup>17</sup> and demonstrate that lensing has a strong effect on observations of the bright SDSS quasars at  $z \gtrsim 5.8$ .

To find the magnification bias more accurately, we have computed the probability distributions  $\frac{dP_{\text{sing}}}{d\mu}$  and  $\frac{dP_{\text{mult}}}{d\mu_{\text{tot}}}$  for the magnification  $\mu$  of randomly positioned singly-imaged sources, and for the sum of magnifications  $\mu_{\text{tot}}$  of randomly positioned multiply-imaged sources due to gravitational lensing by foreground galaxies. We assume that the lens galaxies have a constant co-moving density [as the lensing rate for an evolving (Press-Schechter) population of lenses differs only by  $\lesssim 10\%$  from this case<sup>17</sup>] and are primarily early-type (E/S0) SIS galaxies<sup>18</sup> whose population is described by a Schechter function with parameters<sup>19</sup>  $n_{\star} = 0.27 \times 10^{-2} \text{ Mpc}^{-3}$  and  $\alpha_s = -0.5$ . We assume the Faber-Jackson relation  $(L_g/L_{g\star}) = (\sigma_g/\sigma_{\star})^4$  where  $\sigma_g$  is the velocity dispersion of the lens galaxy, with  $\sigma_{\star} = 220 \text{ km sec}^{-1}$  and a dark matter velocity dispersion that equals the stellar velocity dispersion<sup>18</sup>. We ignore dust extinction by the lens galaxy, which mainly arises in the much rarer spiral galaxy lenses. Potential lens galaxies must not be detectable in the survey data used to select the objects. Galaxies having  $i^* < 22.2$  (around 30% of the potential lens population) are not considered part of the lens population. To compute  $i^*$  for a galaxy having velocity dispersion  $\sigma$  at redshift  $z$ , we use  $L_{g\star}$  from the 2dF early-type galaxy luminosity function<sup>19</sup>, the Faber-Jackson relation, color transformations with a  $k$ -correction<sup>20,21</sup>, and the evolution of the rest-frame B-band mass-to-light ratio<sup>22</sup>.

Our lens model includes microlensing by the population of galactic stars which is modeled as a de Vaucouleurs profile of point masses embedded in the overall SIS mass distribution. The surface mass density in stars for galaxies at  $z = 0$  is normalized so that the total cosmological density parameter of stars<sup>23</sup> equals 0.005. At  $z > 0$  the total mass density in stars is assumed to be proportional to the cumulative star-formation history<sup>24,25</sup>. The parameters of the de Vaucouleurs profiles are taken from a study of the fundamental plane<sup>26</sup>, the microlens mass is chosen as  $0.1M_{\odot}$  and the source size as  $10^{15} \text{ cm}$  (corresponding to ten Schwarzschild radii of a  $3 \times 10^8 M_{\odot}$  black hole). The probabilities  $\frac{dP_{\text{sing}}}{d\mu}$  and  $\frac{dP_{\text{mult}}}{d\mu_{\text{tot}}}$  closely resemble the standard form for the *SIS* and were computed<sup>27</sup> by

combining numerical magnification maps with the distribution of microlensing optical depth and shear along lensed lines of sight, and the distribution  $\left[\tau_{\text{mult}} \frac{dP_{\text{mult}}}{d\mu} + (1 - \tau_{\text{mult}}) \frac{dP_{\text{sing}}}{d\mu}\right]$  was normalized to have unit mean.

The fraction of sources which are multiply imaged due to gravitational lensing is

$$F_{\text{MI}}(z) = \frac{\int_0^\infty d\mu' \tau_{\text{mult}} \frac{dP_{\text{mult}}}{d\mu'} N(> \frac{L_{\text{lim}}}{\mu'}, z)}{\int_0^\infty d\mu' \left[ \tau_{\text{mult}} \frac{dP_{\text{mult}}}{d\mu'} + (1 - \tau_{\text{mult}}) \frac{dP_{\text{sing}}}{d\mu'} \right] N(> \frac{L_{\text{lim}}}{\mu'}, z)}, \quad (3)$$

where  $\tau_{\text{mult}} = 0.0059$ . We find a value of  $F_{\text{MI}} \sim 0.30$  for the color-selected, flux-limited  $z \gtrsim 5.8$  sample. This value is higher by two orders of magnitude than the lens fraction at low redshifts and demonstrates that lensing must already be considered in a sample with only 4 objects. For comparison, our model (including the entire lens population of lens galaxies) predicts  $F_{\text{MI}} \sim 0.01$  at  $z \sim 2$  for  $m_B < 20$ . These calculations do not include selection effects for flux ratios (though the large magnification bias will favor flux ratios near 1) or image separations which may serve to lower  $F_{\text{MI}}$ .

A sub arc-second resolution K-band image has been obtained for the  $z = 5.80$  quasar<sup>1</sup>, and it was found to be an unresolved point source. To our knowledge this is the only  $z \gtrsim 5.8$  quasar for which sub arc-second resolution optical imaging is currently available. However a program to image these quasars with the *Hubble Space Telescope* which will determine the multiple-image fraction, is expected to begin within the coming year (X. Fan private communication). Note though that single image quasars might still be magnified by a factor of  $\sim 2$ . Recent Chandra observations<sup>28</sup> of the  $z = 6.28$  quasar show photons detected on the edge of the extraction ( $1.2''$ ) circle, offering a hint that this quasar may be lensed.

We have computed the distribution of magnifications observed for a sample of quasars brighter than  $L_{\text{lim}}$  at redshift  $z$ ,

$$\frac{dP}{d\mu_{\text{obs}}} = \frac{\left[ \tau_{\text{mult}} \frac{dP_{\text{mult}}}{d\mu} \Big|_{\mu=\mu_{\text{obs}}} + (1 - \tau_{\text{mult}}) \frac{dP_{\text{sing}}}{d\mu} \Big|_{\mu=\mu_{\text{obs}}} \right] N(> \frac{L_{\text{lim}}}{\mu_{\text{obs}})}}{\int_0^\infty d\mu' \left[ \tau_{\text{mult}} \frac{dP_{\text{mult}}}{d\mu} \Big|_{\mu=\mu'} + (1 - \tau_{\text{mult}}) \frac{dP_{\text{sing}}}{d\mu} \Big|_{\mu=\mu'} \right] N(> \frac{L_{\text{lim}}}{\mu'})}. \quad (4)$$

In Figure 2 we show the probability that the magnification of a quasar is higher than  $\mu_{\text{obs}}$ , assuming that the quasar belongs to a sample at a redshift  $z \sim 6$  with the SDSS magnitude limit of  $z^* < 20.2$ . The plotted distribution is highly skewed; the median magnification is  $\text{med}(\mu_{\text{obs}}) \sim 1.2$  while the mean magnification is as high as  $\langle \mu \rangle = 24$ . Thus, one or more of the  $z \gtrsim 5.8$  quasars are likely to be highly magnified while most should be magnified at a low level.

The large values of the magnifications and the highly skewed shape of the distributions in Figure 2 suggest that lensing must alter the observed luminosity function. Indeed, we find that the space density of quasars with  $M_B < -27.6$  is increased by 40%, and that the slope is decreased by 0.15. A quasar magnified by  $\mu$  is detectable to luminosities as low as  $L_{\text{lim}}/\mu$  and has its luminosity,  $L$ , overestimated by  $\mu$ . The factor  $R_{LD}$  by which the luminosity density of quasars brighter than  $L_{\text{lim}}$  is overestimated due to lensing is therefore

$$R_{LD} = \frac{\int_0^\infty d\mu' \left[ \tau_{\text{mult}} \frac{dP_{\text{mult}}}{d\mu'} + (1 - \tau_{\text{mult}}) \frac{dP_{\text{sing}}}{d\mu'} \right] \int_{L_{\text{lim}}/\mu'}^\infty dL' \mu' L' \phi(L')}{\int_{L_{\text{lim}}}^\infty dL' L' \phi(L')} \quad (5)$$

We find  $R_{LD} \sim 2$ , implying that naive computation of the quasar luminosity density from the  $z \gtrsim 5.8$  sample might significantly overestimate its true value. Magnification bias also affects quasars that are not multiply imaged. We therefore predict an enhanced angular correlation on the sky between  $z \sim 6$  quasars and foreground galaxies.

The high source redshifts imply image separations that are slightly larger than usual<sup>17</sup>,  $\sim 1\text{--}2''$ . Furthermore, the lenses are likely to be found at higher redshift<sup>17</sup> due in part to the higher source redshifts but also because bright (low- $z$ ) lenses are excluded. Lensing of high-redshift quasars allows measurement of the masses of the lens galaxies<sup>22</sup> at redshifts higher than currently possible. Furthermore, quasar microlensing should be common over a ten year baseline<sup>27</sup>. This offers the exciting possibility of measuring the source size, and hence black-hole mass, which in turn yields the ratio between quasar luminosity and its Eddington value, a quantity that will be useful in constraining models of structure

formation<sup>3</sup>.

The recently published spectra of the highest redshift quasar <sup>8,29</sup> limit the flux in the Gunn-Peterson<sup>30</sup> trough to less than  $3 \times 10^{-19} \text{ erg sec}^{-1} \text{ \AA}^{-1}$ . Given the importance of this first observation of the re-ionization epoch and of future complimentary observations, we are motivated to ask whether or not lens galaxy light will contaminate deep observations of the Gunn-Peterson trough in the quasar spectrum, even though such a galaxy is not detected in the initial imaging survey. We have computed the flux of lens galaxies at redshift  $z$  with velocity dispersion  $\sigma$ , and convolved the results with the joint probability distribution for the lens galaxy redshift and velocity dispersion. We find that  $\sim 40\%$  of multiple image lens galaxies ( $i^* < 22.2$ ) will contribute flux in the Gunn-Peterson trough above a level of  $3 \times 10^{-19} \text{ erg sec}^{-1} \text{ \AA}^{-1}$ . For some quasars the contamination of the Gunn-Peterson trough by flux from lens galaxies may limit the ability of deep spectroscopic observations to probe the evolution of the neutral hydrogen fraction during the epoch of reionization.

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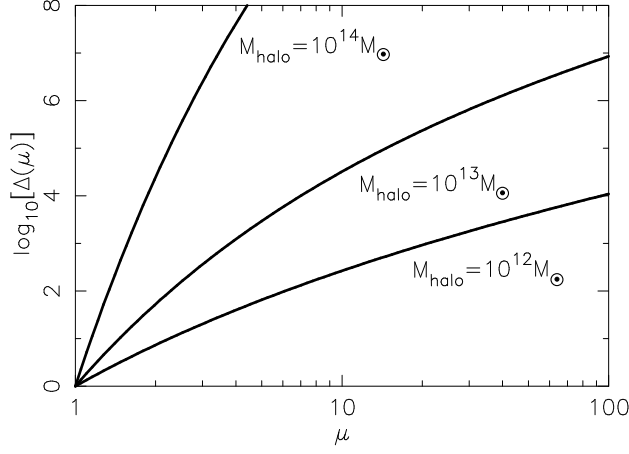


Fig. 1.— Enhancement in the co-moving space density of host galaxies for quasars at  $z = 6$  as a function of the magnification due to lensing,  $\mu$ . Ignoring lensing, the Eddington limit implies<sup>3</sup> a minimum black hole mass of  $\sim 3 \times 10^9 M_\odot$  for the SDSS quasars at  $z \sim 6$ . The inclusion of lensing reduces the implied (minimum) black hole mass by a factor of  $\mu$ . Given some fixed efficiency for assembling gas into a central black hole within a galaxy, the implied mass of the host galaxy is also lowered by  $\mu$  after lensing is included. Plotted is  $\Delta(\mu) = \frac{dn}{d(\log[M/\mu])} / \frac{dn}{d(\log M)}$ , where  $\frac{dn}{d(\log M)}$  is the Press-Schechter<sup>5</sup> comoving density of galaxy halos with mass  $M$  per logarithmic interval in  $M$ . We show curves for three possible halo masses of quasar hosts,  $M_{\text{halo}} = 10^{12} M_\odot$ ,  $10^{13} M_\odot$ , and  $10^{14} M_\odot$ . As seen, the enhancement in the implied abundance of quasar hosts increases with increasing halo mass. The above three choices for halo masses are all *conservatively* lower than inferred for the same black hole mass in the local universe<sup>6</sup>. Throughout this *Letter* we assume the standard cosmological parameters  $\Omega_m = 0.35$ ,  $\Omega_\Lambda = 0.65$ ,  $\Omega_b = 0.05$ ,  $H_0 = 65 \text{ km sec}^{-1} \text{ Mpc}$ ,  $n = 1$  and  $\sigma_8 = 0.8$ .

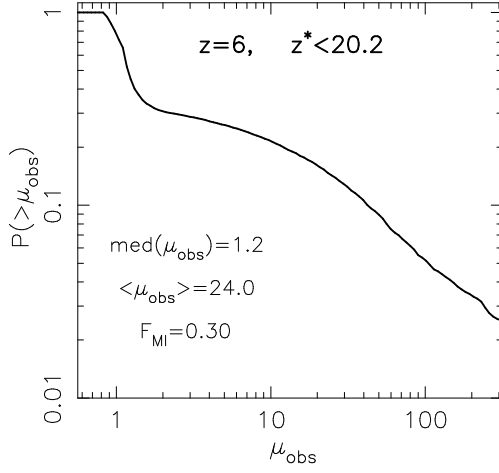


Fig. 2.— The probability of observing a magnification larger than  $\mu_{\text{obs}}$  for a quasar at a redshift  $z = 6$  in a sample with a magnitude limit  $z^* < 20.2$ . The distribution is highly skewed, having a median of  $\text{med}(\mu_{\text{obs}}) = 1.2$  and a mean of  $\langle \mu_{\text{obs}} \rangle = 24.0$ . The multiple image fraction is  $F_{\text{MI}} = 0.30$ . We have also computed the *a-posteriori* values of  $F_{\text{MI}}$  and  $\langle \mu \rangle$  for specific quasars. For SDSS 0836-0054 ( $z = 5.82$ ), SDSS 1306-0356 ( $z = 5.99$ ) and SDSS 1030-0524 ( $z = 6.28$ ) we find  $F_{\text{MI}} = 0.40, 0.32$  and  $0.31$ , and  $\langle \mu \rangle = 50, 25$  and  $23$ , respectively. While Fan et al.<sup>1</sup> find that  $\beta_h = 3.43$  is consistent with the luminosities of the  $z \gtrsim 5.8$  quasars, it is possible that the luminosity function at high redshift is not as steep as  $\beta_h = 3.43$ . In this case, the magnification bias will not be as large and the mean magnification and fraction of quasars that are multiply imaged will be lower. We have re-computed the lens statistics assuming that the bright end slope is significantly flatter at high redshift. Assuming that  $\beta_l = 1.64$  at all redshifts and  $\beta_h = 3.43$  for  $z < 3$  but  $\beta_h = 2.58$  for  $z > 3$  (the value found<sup>12</sup> for quasars at  $z \sim 4.3$ ) we inferred similar parameters to describe the observed luminosity function as before [ $L_\star(0) = 1.5 \times 10^{11} L_\odot$ ,  $z_\star = 1.6$ ,  $\xi = 3.3$ ,  $\zeta = 2.65$ ]. Remarkably, the multiple image fraction is still nearly 0.1 in this case. Additional uncertainty in the calculation arises in the choice of lens model. The value  $\tau_{\text{mult}}$  is proportional to  $n_\star \sigma_\star^4$ . The dependence of  $F_{\text{MI}}$  on  $\tau_{\text{mult}}$  is complex [see equation (3)], however large magnification biases result in a relation that is less sensitive than linear. For example, reducing the value of  $\tau_{\text{mult}}$  by a factor of 1.5 results in  $F_{\text{MI}} = 0.22$  if  $\beta_h = 3.43$  and  $F_{\text{MI}} = 0.043$  if  $\beta_h = 2.58$ .